A line tangent to an ellipse is $y - m\,x = \sqrt{m^2\,a^2 + b^2}$. But

$$a = \frac{c(n+1)}{2}, \text{ and } b = \frac{c(n-1)}{2},$$

$$\text{hence } y - m\,x = \sqrt{\frac{m^2\,c^2\,(n+1)^2}{4} + \frac{c^2\,(n-1)^2}{4}},$$

which after reduction becomes, since $n = \dfrac{z}{a}$,

$$4\,a^2\,m^2\,x^2 - 8\,a^2\,m\,x\,y + 4\,a^2\,y^2 - c^2\,(m^2+1)\,z^2$$
$$-2\,a\,c^2\,(m^2-1)\,z - a^2\,c^2\,(m^2+1) = 0. \;\ldots\ldots(e)$$

This is the tangent cylinder, and its sections are hyperbolas; as for instance that by the plane $y = 0$, for which we have

$$4\,a^2\,m^2\,x^2 - c^2\,(m^2+1)\,z^2 - 2\,a\,c^2\,(m^2-1)\,z - a^2\,c^2\,(m^2+1) = 0.$$

If $m = 0$ in equation (e) it reduces to $2\,a\,y \mp a\,c \pm c\,z = 0$,

$$\text{or } y = \mp \frac{c}{2a}\,z \pm \frac{c}{2},$$

two intersecting planes.

If $m = \infty$ we have similarly $2\,a\,x \pm c\,z \pm a\,c = 0$,

$$\text{or } x = \mp \frac{c}{2a}\,z \pm \frac{c}{2}.$$

Motion of a Sphere on an Inclined Plane.—By Walter Siverly, Oil City, Pa.—Let m = the mass of the sphere, a its radius, k its radius of gyration, β the inclination of the plane, u the coefficient of the dynamical friction between the plane and sphere, R the reaction of the plane on the sphere, s the space passed over by the center of the sphere in the time t, ψ the angle through which it has revolved about its center, and let it be projected down the plain with the velocity v, and at the same time impressed with an angular velocity w. For its motion,

$$m\frac{d^2\,s}{d\,t^2} = m\,g\,\sin\beta - u\,R, \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$m\,k^2\,\frac{d^2\psi}{d\,t^2} = a\,u\,R, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

But $R = m\,g\,\cos\beta$, hence from (1) and (2)

$$\frac{d^2\,s}{d\,t^2} = g\,(\sin\beta - u\cos\beta), \quad \ldots\ldots\ldots\ldots\ldots(3)$$

$$k^2\,\frac{d^2\psi}{d\,t^2} = a\,g\,u\cos\beta. \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Integrating (3) and (4),

$$\frac{ds}{dt} = g\,t\,(\sin\beta - u\cos\beta) + v, \quad \ldots\ldots\ldots\ldots(5)$$

$$\frac{d\psi}{dt} = \frac{a\,u\,g\,t\cos\beta}{k^2} + w. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Integrating (5) and (6), (supposing $s = 0$ and $\psi = 0$ when $t = 0$,)

$$s = \tfrac{1}{2}\,g\,t^2\,(\sin\beta - u\cos\beta) + v\,t, \quad \ldots\ldots\ldots\ldots(7)$$

$$\psi = \frac{a\,u\,g\,t^2\cos\beta}{2\,k^2} + w\,t. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

If the plane be smooth $u = 0$, hence from (6)

$$\frac{d\psi}{dt} = w.$$

The angular velocity is constant, and if $w = 0$

$$\frac{d\psi}{dt} = 0,$$

and from (8) $\psi = 0$; hence a sphere placed on a smooth inclined plane will not *roll* but *slide*, and the foot note, page 35, from Peck's Mechanics, is not correct. See *Walton's Mechanical Problems*, prob. 7, p. 446, 3d Edition.

Note on Right-Angled Triangles.—By Joseph B. Mott, Neosho, Mo.—In any right-angled triangle $A\ B\ C$, $A\ C$ being the base, $C\ B$ the perpendicular and $A\ B$ the hypothenuse, if we put $a^2 + 1 =$ the hypothenuse and $a^2—1 =$ the base, we shall have (Eud 47, I,) $2\,a =$ the perpendicular. Hence if we maintain the above *relation* between the sides, we may assign any value whatever to a, either whole or fractional, and all the sides of the triangle will be rational.

For instance, if $a = 1$, (h, b and p representing respectively the hypothenuse, base and perpendicular) we have

$$p = 2 \times 1 = 2,\ b = 1^2 - 1 = 0,\ h = 1^2 + 1 = 2.$$

If $a = 2$, $p = 2 \times 2 = 4$, $b = 2^2 - 1 = 3$, $h = 2^2 + 1 = 5$.

If $a = 3$, $p = 2 \times 3 = 6$, $b = 3^2 - 1 = 8$, $h = 3^2 + 1 = 10$.

And if $a =$ any other value, as 4, 5, 6, 7, 8, &c., the corresponding sides are found to be 8, 5, 17; 10, 24, 26; 12, 35, 37; 14, 48, 50; 16, 63, 65, &c.

If a be assumed fractional, say $= \frac{5}{4}$, $\frac{5}{3}$ and $\frac{5}{2}$ in succession, we shall have for the sides $\frac{1}{16}$ (41, 9, 43); $\frac{1}{9}$ (34, 16, 30), and $\frac{1}{4}$ (29, 21, 20).